Measuring the first two statistics moments using the Correlator resources

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ABSTRACT

The ALMA telescope is composed of 66 high precision antennas, each antenna having 8 high bandwidth digitizers (4Gsamples/Second). It is a critical task to determine the well functioning of those digitizers prior to starting a round of observations. Since observation time is a valuable resource, it is germane that a tool be developed which can provide a quick and reliable answer regarding the digitizer status. Currently the digitizer output statistics are measured by using comparators and counters. This method introduced uncertainties due to the low amount of integration, in addition to going through all the possible states for all available digitizer time which all resulted in the antennas taking a considerable amount of time. In order to avoid the aforementioned described problems, a new method based on correlator resources is hereby presented.

Keywords: Correlator, Digitizers

1. INTRODUCTION

The electronics within each ALMA antenna is able to instantaneously digitize 8 x 2 GHz bandwidth (4 pairs of 2GHz orthogonal polarized signals). Accomplishing this will require 8 digitizers (named as DG) per antenna.

It is very important to ensure that the first two statistics moments (mean and variance) are within the expected values, otherwise the following problems will arise:

- Big DC component: It is expected that, under a 'normal' condition, not to have a 'big' DC offset after the DG adjustment has been completed. A periodic offset check is still desirable for gaining experience from the operation in the field, making it also a good sanity check. It must be noted that the Walsh sequence will cancel the DC offset in a cross-correlation (Walsh sequence are used during TDM scans).
- Loss of efficiency due to non-optimal DG adjustment. For achieving the maximal Signal to Noise ration, the digitizers' threshold must be adjusted to a level which depends of the correlation scheme (see [1][2]).

Currently the TFB[1] resources are being used for measuring the digitizers' statistics. The TFB card provides the logic resources for counting – during 1milli-second – the amount of samples which match with a pre-defined state. Those logic resources allow getting statistics from the input stage of the TFB (3-bits samples) as well as from the output stage (2-bits samples).

An alternative method is presented within this document which is the idea to use the already available correlator resources for getting the statistical first- and second- moments. This method will allow us to acquire results with less dispersion than the current method for the same or less amount of time.

One possible usage of this approach is to run a quick system sanity check prior to starting an observation project. This method will not replace the existing one, since it does not enable the ability to check artifacts due to a digitizer or DTS[2] malfunctioning.

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[1] TFB: Tunable Filter Card. This is the correlator hardware which gets the samples already digitized.
[2] DTS: Digital Transmission System. This subsystem encompasses FPGAs, Optoelectronics and Fiber Optics.

2. OPERATIONS THEORY

Variance (second statistical moment):
Using the lag0[3] value of the autocorrelation (correlation function for zero time shift) we can determine the variance and therefore the standard deviation.
The auto-correlation is defined as:

Equation 1

$$R_{XX}(\tau) = \sum_{i=-\infty}^{\infty} X(i) \cdot X(i-\tau)$$

And the variance:

Equation 2

$$Var(X) = \sum_{i=1}^{n} P_i \cdot (x_i - \mu)$$

Since the correlation is computed using finite resources, the sum in Equation 1 is computed for a finite number of values of X, and the amount of elements to be summed is named integration time.
The correlator hardware produces what is named *"raw lags"* which is defined as:

Equation 3

$$R_{XX\,raw}(\tau) = \frac{1}{512} \sum_{i=0}^{N} X(i) \otimes X(i-\tau)$$

Scaling by 1/512 comes from the fact that ALMA1 chip[4] is discarding the 9 least significant bits of the addition result.
The operation $\otimes$ presented in Equation 3 is defined as:

	01	00	11	10
01	9	3	-3	-9
00	3	1	-1	-3
11	-3	-1	1	3
10	-9	-3	3	9

Table 1, Multiplication table for 2 bits samples

[3] In this text the meaning of "lag0" is the result of correlate two digitized signal for a time shift equal to zero. The result of the auto-correlation of a random variable for time shift zero is equal to the variance of the same random variable.
[4] ALMA1 chip is the piece of hardware responsible to calculate the correlation function for a given amount of time. This is the digital custom integrated circuit.

	011	010	001	000	111	110	101	100
011	49	35	21	7	-7	-21	-35	-49
010	35	25	15	5	-5	-15	-25	-35
001	21	15	9	3	-3	-9	-15	-21
000	7	5	3	1	-1	-3	-5	-7
111	-7	-5	-3	-1	1	3	5	7
110	-21	-15	-9	-3	3	9	15	21
101	-35	-25	-15	-5	5	15	25	35
100	-49	-35	-21	-7	7	21	35	49

Table 2, Multiplication table for 3 bits samples

Here is where the simplifications start:
The first one comes from the fact that in auto-correlation the only possible product is:
For 2-bits samples $x_i \in \{9,1\}$
And for 3-bits $x_i \in \{49,25,9,1\}$

The second simplification is that the mean value is zero ($\mu = 0$), this means that Equation 2 can be expressed as:

$$Var(X) = \sum_{i=1}^{n} P_i \cdot x_i^2, \quad n = 2 \ for\ 2\ bits\ samples, \quad n = 4\ for\ 3\ bits\ samples$$

And the last simplification, assuming symmetrical distribution, this means:
$P(x_i = L) = P(x_i = -L)$

<u>2-bits case</u>

For the 2 bits case the quantization step $\alpha = \frac{V_{th}}{\sigma}$ [5] which yields the highest efficiency (0.88) is $\alpha = 1$

state	Probability
01	0.1635
00	0.3365
11	0.3365
10	0.1635

Table 3, Probability density for a quantization step equal to 1

Therefore the variance for optimally distributed samples will be (2-bits case):

[5] The symbol α is named quantization step. For reaching the maximal efficiency this parameter must be adjusted. The value of this parameter is what drives the probability values expressed here.

Equation 4

$$Var(X) = \sum_{i=1}^{n} P_i \cdot x_i^2 = 2 \cdot (9 \cdot 0.1635 + 1 \cdot 0.3565) = 3.356$$

<u>3-bits case</u>
For the 3 bits case the highest efficiency (0.96) is met using $\alpha = 0.604$
This means:

state	Probability
011	0.0349931
010	0.0785305
001	0.1593982
000	0.2270782
111	0.2270782
110	0.1593982
101	0.0785305
100	0.0349931

Table 4, Probability density for a quantization step equal to 1

Equation 5

$$Var(X) = \sum_{i=1}^{n} P_i \cdot x_i^2 = 2 \cdot (49 \cdot 0.035 + 25 \cdot 0.0785 + 9 \cdot 0.1594 + 1 \cdot 0.2271) = 10.6792$$

The raw lag value for zero time shift, lag0, and optimally distributed samples is:

Equation 6

$$R_{XX}(0) = \frac{1}{512} \cdot N \cdot Var(X)$$

Where *N* is the amount of samples to be integrated.

Since we can easily obtain the $R_{XX}(0)$ and we also know in advance the value of N (integration time), from Equation 6 we can know the actual value of the variance of X. The variance expected values are also known. See equations 4 and 5.

Therefore the usage of equations 4,5 and 6 will permit to have an idea about how well the digitizers thresholds are adjusted. How those values are combined to get the amplitude error is presented below.

<u>Amplitude error</u>:
The amplitude error will be expressed in units of rms[6] (or σ),
We note that $\sigma = \sqrt{Var(X)}$

[6] Root mean square.

$$error[\%] = 1 - \sqrt{\frac{R_{XX\ meas}}{R_{XX\ theo}}}$$

$$R_{XX\ Theo} = \frac{1}{512} \cdot N \cdot 3.6167\ (2 - bits\ samples)$$

$$R_{XX\ Theo} = \frac{1}{512} \cdot N \cdot 10.6792\ (3 - bits\ samples)$$

DC offset (mean or first statistical moment)
The samples' sum must approach zero; otherwise most of the signal energy will be located in spectral channel zero.
First simplification (or condition): the samples can be characterized as a stationary stochastic process, where the autocorrelation is zero for a time shift different than zero (In a band limited signal this in not true for the time shift near to zero).
Therefore the unbiased correlation for $\tau \neq 0$ must be zero.
For having more samples we can use more than one lag (in other words more than one value of the $R_{XX}(\tau)$.

Aiming at avoiding the effects of the band-limited signal, the values of $R_{XX}(\tau) \quad for \quad 64 \leq \tau \leq 127$ will be averaged (this will avoid to include the side lobes of the autocorrelation function).

Let's name the average of the lags $R_{average} = \frac{1}{64} \cdot \sum_{64}^{127} R_{XX}(i)$

$$error[\%] = k \cdot \sqrt{R_{average}}$$

We are calculating the square root of the lag since the lag is generated by the multiplication of two samples. The *k* constant matches the results of the raw lags to RMS units.
The *k* constant will be evaluated with aim of the simulation results.

3. SIMULATION RESULTS

Excel software has been used for simulating different results. Two sorts of simulation have been generated for testing this method.

One simulation established the relation between the real amplitude error, and what would be produced by the method presented here. The goal of this simulation is to establish the possible results *a priori*. For this simulation it was assumed a mean equal to zero, and different lag0 values were generated for different variances. The obtained results can be depicted in Figure 1.

For simulating the value of the *lag0* for different integration times and different threshold the following formula will be used:

$$lag0 = \frac{1}{512}\left[9 \cdot P(V \le V_{th}) + 5 \cdot P(V_{th} < V < 0) + 5 \cdot P(0 < V < V_{th}) + 9 \cdot P(V_{th} \le V)\right] \cdot N$$

Where N is the amount of samples to be accumulated.
P(X) is the probability density function of associated to the range X (for a Normal distribution)

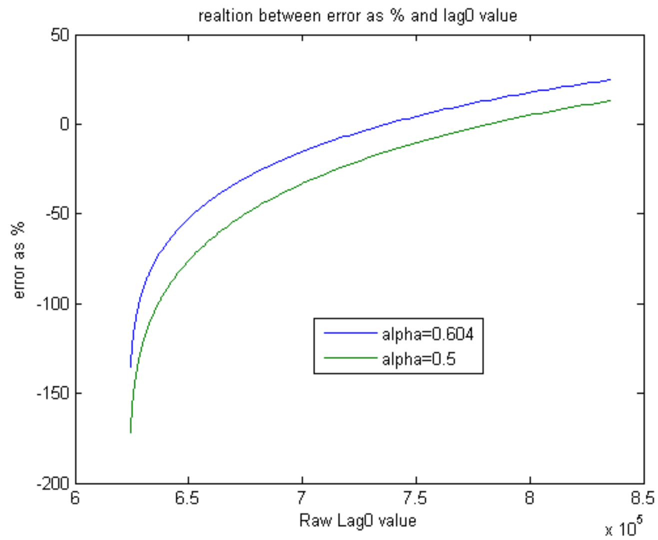

Figure 1, relation between Lag0 values and amplitude errors

The second simulation establishes the relation between the measured DC offset using this method, and the actual value. For this simulation it was assumed a variance equal to the optimal value. The result of the simulation can be seen in Figure 2

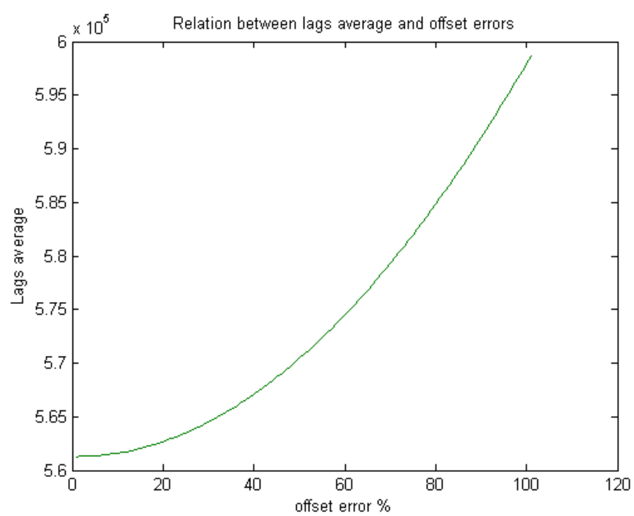

Figure 2, Relation between lags average and offset error.

4. IMPLEMENTATION

The ultimate objective of this development is to have a tool which may quickly provide information about the right tuning of the digitizers associated to a set of antennas.

The DPI[7] LRU[8] will be used for transferring the lag0 value via CAN bus and also calculate the average of 64 lags.

Lag0 will be used for checking the "amplitude" error with respect to the theatrical one (or variance as it has been used within this document).

The average of 64 lags will be used for getting the "offset" error of $R_{XX}(\tau)\ for\ 64 \le \tau \le 127$

Three new CAN protocols have been established for performing this new feature:

- DOWNLOAD_ANTENNA _MASK: Provides a mask to select which antennas are going to be included for getting the lag0 and lag average.
- GET_LAGS_INFO_READINESS: Status of lag 0 and lag average information completeness
- READ_BACK_LAG0_AND_LAGS_AVERAGE: The 128 (32 antennas, 2 polarizations, lag0 + lags average) lags values (4 bytes each) are read back for a total of 1024 bytes.

[7] Data Port Interface, this is the hardware responsible to transfer the lags (or correlation function) to the real time computers responsible to process the produced data.
[8] LRU, Line Replaceable Unit.

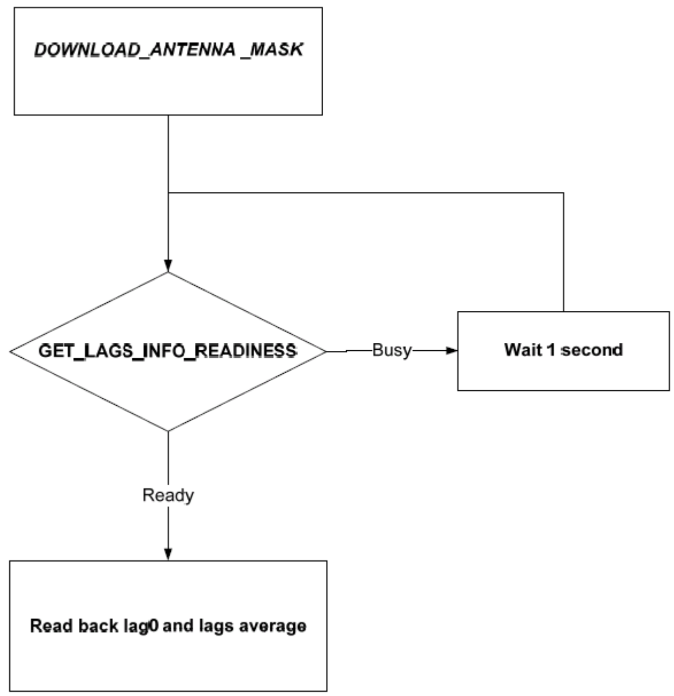

Figure 3, CAN transactions flowchart.

5. PRELIMINARY RESULTS

In order to showcase the efficiency and advantages of this new method, the results presented are focused in a demonstration of accurate results in offset and magnitude error, including a timing analysis. The two statistical moments obtained from this approach have been developed and tested on 2-Antenna correlator in OSF (TFINT) and later tested on each quadrant of the 64 antennas Baseline Correlator.

5.1 First statistical moment

First, the offset error (mean or first statistical moment) for the current TFB samples retrieval is compared with the offset obtained from Lag Average results taken directly from auto-correlation output.

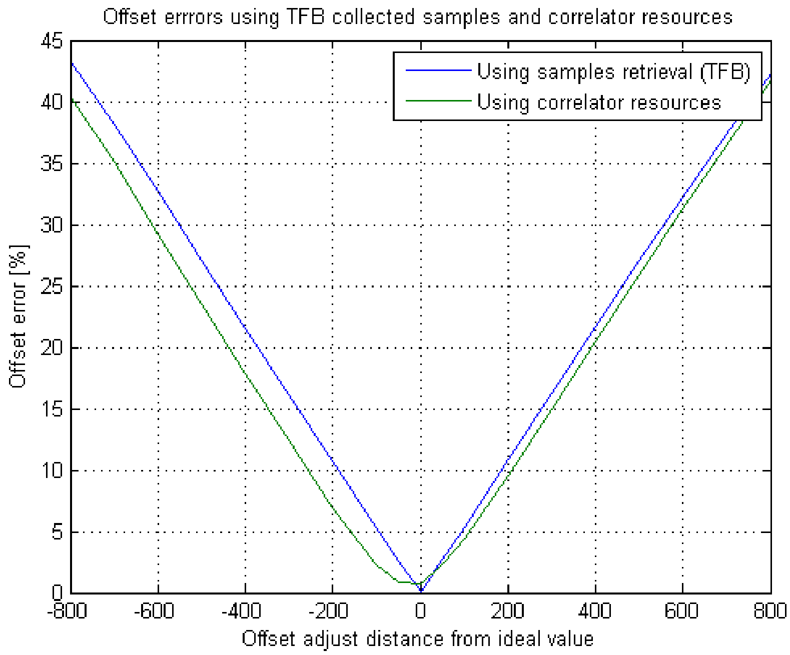

Figure 4, Offset errors using TFB Samples and Correlator Resources.

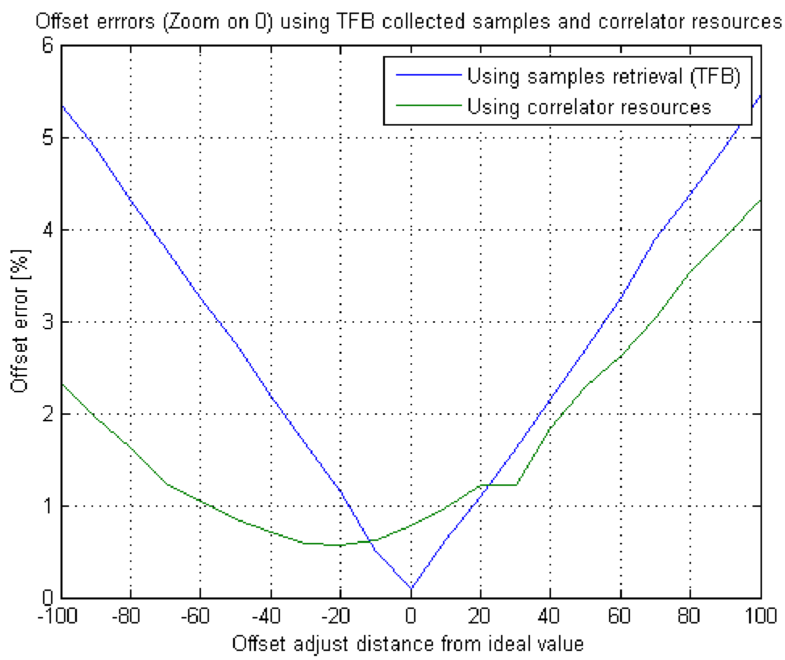

Figure 5, Zoom-in ideal offset value.

As can be appreciated in Figure 4, the offset error using correlator resources follows the trend as the 'samples version' does. This graph is shown using absolute values to establish a common ground since the correlator output version only gives positive offset error values.

A close up near the digitizer best offset value adjust is presented in Figure 5. According to this illustration it can be seen that the best offset error for ‘samples’ version is not necessarily in the same position for ‘correlator resources’ version. This curve shift from abscissa axis can also be deduced in Figure 6 where the offset error distance between these approaches is high (~4.5%) at a negative offset adjust in digitizer, and low (~2%) at positive offset adjust in digitizer.

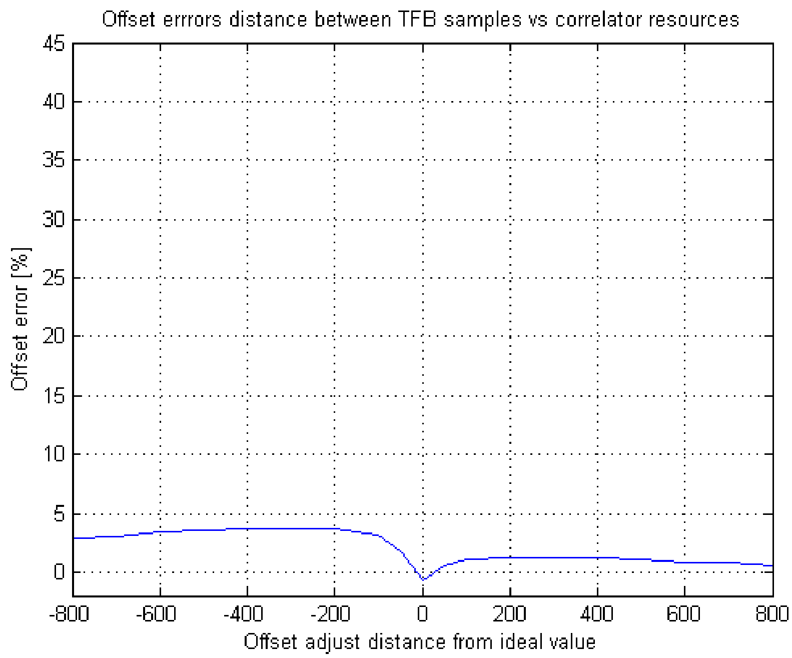

Figure 6, Offset error difference along digitizer adjust.

5.2 Second statistical moment

For exercising the method presented here and compares it against the previous method (measuring the statistics directly), the following setup was made:

Changing the attenuation of the input IF signal which goes to the analog digital converter different variance values were generated, for each one it was measured the amplitude error (or how far the samples’ variance were from the optimal value)

Due to the hardware limitations only 9 values were obtained.

Using samples retrieval	Using correlator resources
-25.75	-24.90
-19.76	-20.53
-11.98	-13.49
-7.18	-8.39
-1.2	-2.17
+2.7	1.21
+7.22	5.81
+10.89	9.63

Table 5

As can be seen in table 5, the amplitude errors using correlator resources follows the trend as the 'samples version' does.

An important factor to consider when a preliminary test is performed during the valuable observation block is processing time. This is due to the amount of calculation and data transfer operations required depends on the algorithm used and the amount of antennas used. Figure 5 exhibits such features where there is a high contrast between both approaches since the 'TFB samples' version queries repeated data transactions over the CAN bus for a single antenna check. On the other hand, 'correlator resources' version receives a single block of data for each DPI (2 per quadrant, 8 in total). Furthermore, it is worth mentioning that this data transaction is not dependent on the number of antennas being checked.

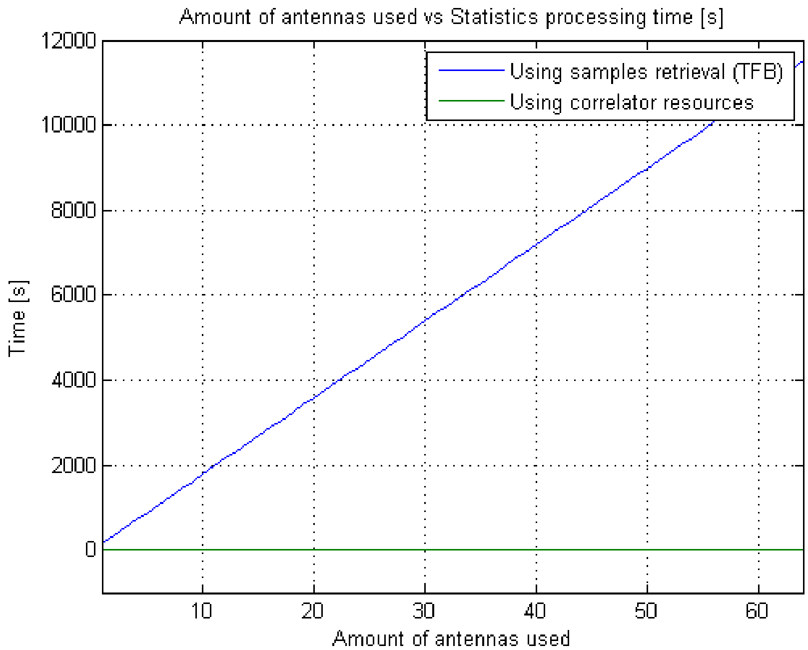

Figure 7, Algorithms processing time as function of the amount of antennas used.

The developed tool meets all the specifications and can be used to predict wrong digitizer configurations in a precise and fast manner for a single antenna or a full array of 64 antennas, all baseband pairs and polarizations. The work presented here is an excellent candidate to be included in future observation runs.

Finally as a validation, the results are more than 0.99 correlated to what was predicted in the simulations.

6. CONCLUSIONS

The preliminary results have demonstrated a concordance between the results provided by the old method and the new one presented herein.

However it is important to highlight that the right way to use this method is:

- First check if the first moment or mean is within an acceptable range (less than 5% of error respect to the ideal value)
- If the first statistical moment is out of an acceptable value, the value of the second statistical moment will be meaningless.

This method is not suitable for addressing problems such as the following:

- Periodic spikes.
- Artifacts in the frequency domain

As a quick verification of the digitizers' proper tuning, this new tool has demonstrated to be effective. Therefore, the established goal for this tool has been achieved.

This method will allow us to verify in a quick manner the tight performance of the digitizers and save time for detecting arising issues and do not wait until reduce the data.

7. FUTURE IMPROVMENTS

The calculation of the first two statics moments can be accelerated by using the available FPGA resources. Currently the average of the lags values is being calculated by the microcontroller, which does not run as fast the FPGA.

The band-pass response can be also considered at the time of the lags average calculation. This will allow to use a larger set the values.

8. REFERENCES

[1] Cooper, B.F.C [Correlators with two-bit quantization], Aust. J. Phys. 23-51 (1970).
[2] Thompson, A.R [Quantization Efficiency for Eight or more Sampling Leveles], MMA memo 220 (1998)